IHS Economics Series
Working Paper 110
February 2002

Investment and Current Account Dynamics in an Open Economy Status Seeking Framework

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Impressum

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Title:

Investment and Current Account Dynamics in an Open Economy Status Seeking Framework

ISSN: Unspecified

2002 Institut für Höhere Studien - Institute for Advanced Studies (IHS)

Josefstädter Straße 39, A-1080 Wien

E-Mail: office@ihs.ac.at

Web: www.ihs.ac.at

110

Reihe Ökonomie

Economics Series

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Abstract

In this paper we analyze the implications of status-preference, modeled as relative wealth, for the current account in a small open economy framework with capital stock dynamics. We demonstrate that the transitional dynamics of the economy is characterized by two distinct speeds of adjustment: a speed of adjustment arising from status-preference and a speed of adjustment arising from installation costs of investment. This structure implies that the current account balance depends on both speeds of adjustment as well as on the long-run equilibrium, which is a function of the degree of status-consciousness. As a consequence, the current account can exhibit non-monotonic behavior in transition to the steady-state equilibrium.

Keywords

Current account, status seeking, relative wealth, open economy dynamics

JEL Classifications

E21, F41

Comments

This working paper is a modified version of *Economics Series* paper No. 107, which appeared in October 2001. It has been extended in this version. The author wishes to thank Franz X. Hof for his very helpful comments. The author also acknowledges the generous financial support of the Oesterreichische Nationalbank (OeNB) (Jubilaeumsfondsprojekt No. 8701).

Contents

1. Introduction

A well-known and problematic characteristic of the small open economy, representative agent Ramsey model is the fact that consumption and the stock of assets do not, under the assumption of perfect capital mobility, converge to "sensible" levels if the domestic rate of time preference differs from the world interest rate. For instance, if the rate of time preference exceeds the world interest rate, agents have the incentive to mortgage all their human and nonhuman wealth, with consumption converging to zero. On the other hand, if the economy "patient" in the sense that its rate of time preference is less than the world interest rate, then it will accumulate wealth to the point that it eventually "stops" being a small open economy.[1] A solution to this problem is found in a recent paper by Hof and Wirl (2001), who assume that representative agents have preferences not only over own consumption, but also over relative wealth. In this formulation, relative wealth serves as a proxy for an individual's status in society.[2] In their Ramsey model of relative wealth, Hof and Wirl (2001) show that the small open economy—given certain restrictions on parameter values—has an interior long-run equilibrium and saddlepoint stable dynamics.[3] The relative wealth specification has also been used by researchers such as Corneo and Jeanne (1997), Rauscher (1997a), and Futagami and Shibata (1998) to model—in a closed economy framework—the effects of status-preferences on variables such as the rate of economic growth. An alternative approach of modelling status in a dynamic framework is the relative consumption framework, which has been used by Galí (1994), Persson (1995), Harbaugh (1996), Rauscher (1997b), Grossmann (1998), Ljungqvist and Uhlig (2000), and Fisher and Hof (2000). While the relative consumption approach is an appropriate one for certain problems and, furthermore, has a conceptual advantage in the sense that it is easier to "observe" another person's level of consumption, it has the disadvantage that it is incapable—since it does not affect the domestic rate of return on assets—of resolving the issue mentioned at the start of this paper: the existence of an interior equilibrium if

[1] A recent discussion of this phenomenon is found in Barro and Sala-i-Martin (1995), chapter 3.

[2] See Duesenbery (1949), Scitovsky (1976), Hirsch (1976), Boskin and Sheshinski (1978), Layard (1980), and Frank (1985a,b) for general analyses of the implications of a preference for status in economic decision-making.

[3] See Turnovsky (1997) for a detailed discussion of alternative approaches of generating saddlepath transitional dynamics in the small open economy context. Hof and Wirl (2001) also explore the possibility of multiple equilibria in their model.

the rate of time preference differs from the world interest rate. Indeed, the only way to obtain an interior equilibrium in the relative consumption context is to impose equality between the time rate of preference and the world interest rate.[4] We believe, therefore, that the relative wealth approach, which we will adopt here, is a promising one to use to investigate the role played by status-preference in the evolution of small open economies.

One counter-intuitve property of the small open economy Ramsey model that is not, however, eliminated by the introduction of relative wealth preferences is the fact that the domestic capital stock—pinned-down by the exogenous world interest rate—does not possess transitional dynamics. As is well-known, capital stock dynamics in the small open economy context can be restored by assuming that domestic physical investment takes place subject to convex installation costs. This approach has been used by, among others, Brock (1988), Sen and Turnovsky (1989a, 1989b, 1990), and Frenkel, Razin, and Yuen (1996). Assuming standard restrictions regarding the production function, the capital stock and its shadow price (Tobin's q) then possess saddlepoint stable dynamics. The goal of this paper is to investigate the behavior of overall asset accumulation in a model that features both relative wealth preferences and capital stock dynamics. To do so, we will employ a status-preference framework similar to that used by Futugami and Shibata (1998) and Hof and Wirl (2001), together with a standard quadratic installation cost function. After solving for the intertemporal equilibrium of the model, we will show that the small open economy possesses two rates of stable dynamic adjustment: i) a "consumption-side" rate of adjustment that depends on status-preference and ii) a "production-side" rate of adjustment that depends on the characteristics of the installation cost function. It will be case that the dynamics of consumption itself depends solely on the consumption-side rate of adjustment, while the rate of adjustment of domestic physical capital and its shadow price is a function only of the production-side rate of adjustment. In contrast, we will demonstrate that the evolution of the current account balance—corresponding to the accumulation of net financial assets—will depend on both (stable) rates of adjustment. This introduces ambiguities in the dynamics of the current account. In particular, for given initial stocks of net financial assets and physical capital, the path taken by the current

[4] In this case, equilibrium consumption and its shadow value always correspond to their implied steady-state values and, consequently, possess no transitional dynamics.

account need not be monotonic. For example, the current account balance can initially deteriorate, reach zero, and then improve in the transition to steady-state equilibrium. We will show that the factors that determine current account dynamics in our model include the relative magnitudes of the two speeds of adjustment and the long-run equilibrium of the economy, which is a function, in part, of how "strongly" agents value status.

The paper has the following organization. Section 2 describes our model of identical consumer-producers who have preferences over relative wealth and accumulate physical capital subject to installation costs. In this section, we calculate the necessary optimality conditions and derive the resulting macroeconomic equilibrium. This yields differential equations for consumption (the Euler equation), the current account balance, domestic physical capital, and the shadow price of investment. A crucial feature of our model is the assumption that identical agents make the same choices, which yields a symmetric intertemporal equilibrium. In the next section, section 3, we analyze the steady state of the economy and discuss its relationship to status preference. In this context, we show that the long-run levels of net assets and consumption are larger the "stronger" is the preference for relative wealth. We next solve for the economy's linearized dynamics and show that the solutions for the capital stock and its shadow value are derived independently from the Euler equation and the current account balance. The solutions for the capital stock and its shadow value are then used to calculate the paths of consumption and net financial assets. The key implication of this analysis is that the current account balance depends on the speeds of adjustment of both the production and consumption sectors of the economy. This result implies that the path taken by the current account balance in transition to steady-state equilibrium can be non-monotonic, e.g., the current account can deteriorate prior to its eventual improvement. This reflects the fact that the adjustment of the consumption and production sectors of the economy exercise opposing influences on the dynamics of the current account balance. Whether the influence of the consumption or the production sectors dominates during the initial phase of adjustment depends, in part, on how important are relative wealth considerations. In section 3 we will supplement our analysis with numerical illustrations in order to clarify our results regarding the role played by status preference in the determining the intertemporal equilibrium. We conclude the paper with brief remarks in section 4.

2. The Model

We start by assuming a small open economy is populated by a large number of identical, infinitely-lived consumer-producers. Without loss of generality, we specify that the population of consumer-producers is constant. As a consumer, we assume that each agent possesses a general instantaneous utility function over own consumption, c, and status, s, $U = U(c, s)$, that has the following properties:

$$U_c > 0, \quad U_s > 0, \quad U_{cc} < 0, \quad U_{ss} \leq 0, \quad U_{cc}U_{ss} - U_{cs}^2 \geq 0, \tag{1a}$$

$$U_{sc}U_c - U_sU_{cc} > 0, \tag{1b}$$

$$\lim_{c \to 0} U_c(c,\, s) = \infty, \qquad \lim_{c \to \infty} U_c(c,\, s) = 0. \tag{1c}$$

According to (1a), the representative agent as a consumer derives positive, though diminishing, marginal utility from own consumption and positive and non-increasing marginal utility from status. In addition, the utility function U jointly concave, according to (1a), in c and s. Condition (1b) imposes normality on preferences. In other words, this means that the marginal rate of substitution of status for consumption, U_s/U_c, depends positively on c. The next condition, condition (1c), describes the limiting behavior of the marginal utility of consumption. As we outlined in the introduction, we specify that an individual's status—described by the function $s = s(a, A)$—depends on both own net assets (= nonhuman wealth), a, and average net assets of the private sector, A. We assume that the status function is defined for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$. The small open economy's stock of net assets consists of the net stock of loans N owed by the rest of the world and the domestic physical capital stock K, so that $A = N + K$. Correspondingly, own, or individual, net assets equal $a = n + k$. In addition, we assume for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$ that status increases in own wealth, decreases in average wealth, and that the marginal status in own assets is non-increasing, i.e.:

$$s_a > 0, \quad s_A < 0, \quad s_{aa} \leq 0. \tag{2a}$$

We will assume that the status function (2a) takes the following simple ratio form:

$$s(a, A) \equiv \frac{a - \bar{a}}{A - \bar{a}} \tag{2b}$$

Equation (2b) states that own and average wealth are measured with respect to a "lower bound", or "minimum value", which we denote by $\bar{a}$. Since we do not want to rule-out at the outset the possibility that the economy reaches a long-run equilibrium with a negative stock of net assets, we will follow Hof and Wirl (2001) and specify that $\bar{a}$ takes on a negative value, i.e., $\bar{a} < 0$. The parameter $\bar{a}$ can be interpreted as an indicator of domestic residents' aversion to (or tolerance for) indebtedness.[5] It is easily verified that (2b) satisfies the properties given in (2a), for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$.[6]

To help clarify our subsequent analysis, we will parameterize the instantaneous utility function $U(c, s)$. Specifically, we will employ a version of the parameterization used by Futugami and Shibata (1998)

$$U(c, s) = (1 - \theta)^{-1} \left[\left(c^{\eta} s^{\beta} \right)^{1-\theta} - 1 \right], \tag{3}$$

where $\eta > 0$, $\beta > 0$, $\theta > 0$, $1 - \eta(1 - \theta) > 0$, $1 - \beta(1 - \theta) \geq 0$, $1 - (\beta + \eta)(1 - \theta) \geq 0$.[7] Our assumptions ensure that (3) satisfies the restrictions stated above in conditions (1a)-(1c).

As indicated above, we assume that consumer-producers as savers (borrowers) accumulate domestic physical capital k and lend net financial assets (acquire net debts) n in the international credit market. In a fully integrated world capital market, the rate of return on net loans, or financial assets, equals the exogenous and time-invariant world

[5] In our numerical parameterization of the economy described below, we will specify that agents debt-tolerance equals 50% of steady state output.

[6] Hof and Wirl (2001) employ a more general specification of the status function (2b) equal to:

$$s(a, A) \equiv \varphi \left(\frac{a - \bar{a}}{A - \bar{a}} \right); \quad \bar{a} < 0, \quad \varphi > 0, \quad \varphi' > 0, \quad \varphi'' \leq 0.$$

[7] The Futagami and Shibata (1998) specification is recovered by setting $\bar{a} = 0$.

interest rate r^*. In addition, the representative agent inelastically supplies one unit of labor. Consumer-producers then combine capital and labor to produce per-capita output y using a standard per-capita production function equal to $\epsilon \cdot f(k)$, where ϵ denotes total factor productivity, and $f(k)$ satisfies the standard neoclassical properties, which are given by $f'(k) > 0$, $f''(k) < 0$, $f(0) = 0$, $f(k) \to \infty$ as $k \to \infty$, along with the usual Inada conditions.[8] Physical investment, denoted by i, is subject to installation costs that are described by the following constant returns to scale, quadratic function $\Psi(i,k)$

$$\Psi(i,k) = i\left[1 + \frac{h}{2}\frac{i}{k}\right], \quad h > 0 \tag{4}$$

where the parameter h scales the marginal cost of physical investment. Given the economy's technological and financial market possibilities, net financial asset and physical capital accumulation obey the following relationships:

$$\dot{n} = \epsilon f(k) + r^* n - i\left[1 + \frac{h}{2}\frac{i}{k}\right] - c, \tag{5a}$$

$$\dot{k} = i. \tag{5b}$$

Employing an infinite horizon, perfect foresight framework, the agent's maximization problem is formulated as follows

$$\max \int_0^\infty \left\{(1-\theta)^{-1}\left[\left(c^\eta s^\beta\right)^{1-\theta} - 1\right]\right\} e^{-\rho t}\, dt, \quad \rho > 0$$

subject to the flow constraints (5a) and (5b), the initial conditions $n(0) = n_0$ and $k(0) = k_0$, and the No-Ponzi-Game (NPG) restriction, equal to $\lim_{t\to\infty} ae^{-r^* t} \geq 0$. In addition, the parameter ρ denotes the exogenous rate of time preference and s is given by equation (2b). Regarding the rate of time preference, we will assume that agents are "impatient" in the sense that rate of time preference ρ exceeds the world interest rate r^*, $(\rho - r^*) > 0$.[9] A crucial feature of this optimization problem is that the representative agent takes the time

[8] Subsequently, we will simply call per-capita output, "output."

[9] We will use this assumption below to establish that the dynamic system $(\dot{n}, \dot{c})$ possesses an interior, saddlepoint equilibrium.

path of average wealth, $A = N + K$, as given. In other words, each individual is small enough to neglect his own contribution to the average level of wealth. The current value Hamiltonian for this problem corresponds to

$$H(c, n, k, \lambda, q) = (1-\theta)^{-1}\left[\left(c^{\eta}\left(\frac{n+k-\bar{a}}{N+K-a}\right)^{\beta}\right)^{1-\theta} - 1\right]$$

$$+\lambda\left[\epsilon f(k) + r^{*}n - i\left[1 + \frac{h}{2}\frac{i}{k}\right] - c\right] + q'i,$$

where λ is the current shadow value of wealth and $q' = q\lambda$ is the shadow price of domestic capital in terms of foreign assets. The necessary conditions for an interior optimum, $H_c = 0$, $H_i = 0$, $\dot{\lambda} = \rho\lambda - H_n$, and $\dot{q}' = \rho q' - H_k$ are then expressed as

$$\eta c^{\eta-1}s^{\beta}\left(c^{\eta}s^{\beta}\right)^{-\theta} = \lambda, \tag{6a}$$

$$i = \frac{(q-1)}{h}k, \tag{6b}$$

$$\dot{\lambda} = (\rho - r^{*})\lambda - \beta\frac{c^{\eta(1-\theta)}s^{[1-\beta(1-\theta)]}}{N+K-\bar{a}}, \tag{6c}$$

$$\dot{q}' = \rho q' - \lambda\left[\epsilon f'(k) + \frac{h}{2}\left(\frac{i}{k}\right)^{2}\right] - \beta\frac{c^{\eta(1-\theta)}s^{[1-\beta(1-\theta)]}}{N+K-\bar{a}} \tag{6d}$$

where $s \equiv (a - \bar{a}/A - \bar{a})$. Using the necessary conditions (6a) and (6c) and the definition $q' = q\lambda$, we can rewrite the necessary condition (6d) for domestic capital as

$$\frac{\epsilon f'(k)}{q} + \frac{(q-1)^{2}}{2hq} + \frac{\beta c}{q\eta(N+K-\bar{a})s} + \frac{\dot{q}}{q} = r^{*} + \frac{\beta c}{\eta(N+K-\bar{a})s} \tag{6e}$$

where we have substituted for the optimality condition for investment (6b) to derive the left-hand-side of (6e). Observe, moreover, that the condition (6e) describes the equality between the rates of return of domestic capital and net loans. The rate of return of domestic capital—the left-hand-side of (6e)—includes four components. The first term is the marginal physical product of capital in terms of the domestic shadow price of capital q. The second term represents reduction in installation costs arising from an additional unit of capital. The third term is the gain in utility—expressed in terms of output—from higher status, generated by an additional unit of capital, while the fourth term, equal to $\dot{q}/q$, represents the capital gain (or loss) from holding physical assets. The rate of return of net financial assets—the right-hand-side of (6e)—is the sum of the exogenous world interest rate and the utility pay-off in terms of output of accumulating "status-enhancing" net financial assets. Note that the status gain of accumulating domestic physical capital is measured in terms of its shadow price q, while that of net financial assets is scaled by the unitary price of output. The assumptions made above in (1a), (2a)-(2b), and (4) ensure that the Hamiltonian is jointly concave in the control variables c and i and the state variables n and k. This implies that if the limiting transversality conditions $\lim_{t\to\infty} \lambda n e^{-\rho t} = \lim_{t\to\infty} \lambda k e^{-\rho t} = 0$ hold, then the necessary conditions (6a-e) are sufficient for optimality.

We next derive the intertemporal macroeconomic equilibrium. In common with most of the research analyzing the effects of status preference in the dynamic macroeconomic models, we confine our analysis to symmetric equilibria in which identical agents make identical choices. Consequently, $(n+k) = (N+K)$ holds $\forall t \geq 0$. As indicated, $(n+k)$, the net assets, or wealth, of the domestic private sector, consists of physical capital and net financial claims on the rest of the world. Substitution of $(n+k) = (N+K)$ into the optimality conditions (6a)-(6c), (6e), and net financial asset accumulation equation (5a)—where the latter corresponds to the current account balance—results in the following symmetric open economy equilibrium in which the variables (c, i, λ, k, q, n) obey the following relationships

$$\eta c^{-[1-\eta(1-\theta)]} = \lambda, \tag{7a}$$

$$i = \dot{k} = \frac{(q-1)}{h} k, \tag{7b}$$

$$\dot{\lambda} = (\rho - r^*)\,\lambda - \beta \frac{c^{\eta(1-\theta)}}{n + k - \bar{a}}, \tag{7c}$$

$$\dot{q} = r^* q - \epsilon f'(k) - \frac{(q-1)^2}{2h} + \frac{(q-1)\beta c}{\eta(n + k - \bar{a})} \tag{7d}$$

$$\dot{n} = \epsilon f(k) + r^* n - \frac{(q^2 - 1)}{2h} k - c, \tag{7e}$$

as well as the initial conditions $n(0) = n_0$, $k(0) = k_0$ and the limiting transversality conditions $\lim_{t\to\infty} \lambda n e^{-\rho t} = \lim_{t\to\infty} \lambda k e^{-\rho t} = 0$.[10]

We will analyze the properties of the dynamic system (7a)-(7e) in terms of the control variable consumption c, rather than in terms of the shadow of the (current) shadow value of wealth, λ. Taking the time derivative of (7a) and substituting the resulting expression into (7c) results, after division by (7a), in the following modified Euler equation

$$\dot{c} = \sigma^e c[r^e(r^*, c, a) - \rho], \tag{8}$$

where the *effective* elasticity of intertemporal substitution σ^e and the *effective* domestic, or internal, rate of return on assets r^e are given, respectively, by

$$\sigma^e \equiv [1 - \eta(1-\theta)]^{-1}, \qquad r^e(r^*, c, n, k) \equiv r^* + \frac{(\beta/\eta)\,c}{(n + k - \bar{a})} \tag{9a}$$

where:

$$\frac{(\beta/\eta)c}{(n + k - \bar{a})} = \frac{U_s(c,1)}{U_c(c,1)} s_a(a,a) = \frac{U_s(c, s(a,A))}{U_c(c, s(a,A))} s_a(a,A) \mid_{a=A} \tag{9b}$$

The effective rate of return r^e equals the sum of the world interest rate r^* and the marginal rate of substitution of own net assets $a = (n + k)$ for consumption c as perceived by the domestic consumer-producer in a symmetric state in which $(n + k) = (N + K)$ holds.

[10] Hereafter, lower–case variables refer to their aggregate counterparts. An additional implication of the symmetric equilibrium is that the flow of instantaneous utility is independent, since $U = U(c, 1)$, of the stock of net assets.

In this formulation, the symmetric MRS of net assets for consumption represents the additional return to saving due to status preference in which the incremental flow of utility from an extra unit of savings is converted into equivalent units of the consumption good. Observe from (9b) that the symmetric MRS is the product of the MRS of status for consumption in the symmetric equilibrium, given by $[U_s(c,1)/U_c(c,1)] = (\beta/\eta)c$, and the marginal status of own assets in the symmetric equilibrium, equal to $s_a = (a - \bar{a})^{-1} = (n + k - \bar{a})^{-1}$. Since the MRS of status for consumption is a positive function of the parameter β—the utility "weight" of status in the instantaneous utility function (3)—we can refer to β as a measure of the "importance" of net asset accumulation in the quest for status. Differentiating the expression for $r^e(r^*, c, n, k)$ in (9a) with respect to c, n, and k, it is straightforward to show that the symmetric MRS, and hence the effective rate of return, is a positive function of consumption and a negative function of net assets. For our purposes, the advantage of the Euler equation (8) modified for status-preference is that consumption (potentially) displays saddlepath dynamics, which implies that the small open economy (potentially) avoids the counter-intuitive transitional dynamics described in the introduction.[11]

3. Steady State Equilibrium and Macroeconomic Dynamics

3.1. Properties of the Steady State and the Dynamic System

Letting $\dot{c} = \dot{q} = \dot{k} = \dot{n} = 0$ in equations (7b), (7d), (7e), and (8), we derive the following steady-state equilibrium:

$$\frac{(\beta/\eta)\,\tilde{c}}{\tilde{n} + \tilde{k} - \bar{a}} = \rho - r^*, \tag{10a}$$

$$\tilde{q} = 1, \tag{10b}$$

[11]The introduction of the effective rate of return implies that the transversality condition can be written as:

$$\lim_{t\to\infty} \left\{ a\,(t) \exp\left[-\int_0^t r^e(r^*, c(v), n(v) + k(v))\, dv \right] \right\} = 0.$$

Because we focus on long-run equilibria in which $(n + k)$ assumes finite values, this modified transversality condition implies that the NPG holds with equality.

$$\epsilon f'(\tilde{k}) = r^*, \tag{10c}$$

$$\epsilon f(\tilde{k}) - \tilde{c} = -r^* \tilde{n}. \tag{10d}$$

where $\tilde{x}$ denotes the steady state value of a variable x. The steady-state equilibrium described by (10a)-(10d) determines $(\tilde{c}, \tilde{n}, \tilde{q}, \tilde{k})$ and has the following characteristics. Equation (10a) is the steady-state version of the Euler equation, while (10b) indicates that the steady-state shadow value of domestic capital equals unity. According to (10c), the long-run marginal physical product of capital equals the exogenous world interest rate, which determines the long-run values of the capital stock $\tilde{k}$ and output $\tilde{y}$. Finally, because the current account balance is zero in long-run equilibrium, equation (10d) implies that the steady-state excess of output over consumption corresponds to net interest service. A key characteristic of this long-run equilibrium is that the steady-state values of the "production-side" of the economy, i.e., $\tilde{q}$, $\tilde{k}$ and $\tilde{y}$, are determined solely by the steady-state relationships (10b)-(10c) and, consequently, are independent of (10a)-(10d), which we can conveniently term the steady-state "consumption-side" of the economy. Implicitly, solving the steady-state marginal condition (10c) for $\tilde{k}$, we can express the steady-state capital stock as a function of total factor productivity ϵ and the world interest rate r^*:

$$\tilde{k} = \tilde{k}(\epsilon, r^*), \qquad \partial\tilde{k}/\partial\epsilon = -\, f'(\tilde{k})[\epsilon f''(\tilde{k})]^{-1} > 0, \qquad \partial\tilde{k}/\partial r^* = [\epsilon f''(\tilde{k})]^{-1} < 0. \tag{11}$$

According to (11), the steady-state domestic capital stock $\tilde{k}$ (and output $\tilde{y}$) is a positive function of total factor productivity and a negative function of the world interest rate. Substitution of $\tilde{k} = \tilde{k}(\epsilon, r^*)$ into (10a) and (10d) yields the two-equation system that determines the steady-state values of consumption and net financial assets:[12]

$$\frac{(\beta/\eta)\,\tilde{c}}{\tilde{n} + \tilde{k} - \bar{a}} = \rho - r^*, \tag{12a}$$

[12]Hereafter, the functional dependence of $\tilde{k}$ (and $\tilde{y}$) on ϵ and r^* will be assumed.

$$\epsilon f(\tilde{k}) - \tilde{c} = -r^* \tilde{n}. \tag{12b}$$

Equations (12a)-(12b) can then be jointly solved for $\tilde{c}$ and $\tilde{n}$:

$$\tilde{c} = \frac{(\rho - r^*)\, r^*}{\rho - [1 + (\beta/\eta)] r^*} \left[\frac{\epsilon f(\tilde{k})}{r^*} - (\tilde{k} - \bar{a}) \right], \tag{13a}$$

$$\tilde{n} = \frac{1}{\rho - [1 + (\beta/\eta)] r^*} \left[(\beta/\eta)\, \epsilon f(\tilde{k}) - (\rho - r^*)\, (\tilde{k} - \bar{a}) \right]. \tag{13b}$$

Using (11) and (13b), we can also determine the expression for the steady-state *excess* of domestic net assets over its lower bound, i.e., $\left[(\tilde{n} + \tilde{k}) - \bar{a} \right] = (\tilde{a} - \bar{a})$:

$$\left[(\tilde{n} + \tilde{k}) - \bar{a} \right] = (\tilde{a} - \bar{a}) = \frac{(\beta/\eta)\, r^*}{\rho - [1 + (\beta/\eta)] r^*} \left[\frac{\epsilon f(\tilde{k})}{r^*} - (\tilde{k} - \bar{a}) \right]. \tag{13c}$$

We will restrict our attention to long-run equilibria with positive levels of consumption and a positive excess of domestic net assets over its lower bound, i.e., $\tilde{c} > 0$ and $\left[(\tilde{n} + \tilde{k}) - \bar{a} \right] = (\tilde{a} - \bar{a}) > 0$. This requires that we impose the following restrictions on (13a) and (13c):

$$\rho - [1 + (\beta/\eta)] r^* > 0, \quad \frac{\epsilon f(\tilde{k})}{r^*} - (\tilde{k} - \bar{a}) > 0. \tag{14}$$

The first condition of (14) imposes—in effect—an upper bound on the status-preference parameter β for given values of ρ, r^*, and η. The second condition of (14) requires that the present discounted value of output exceed the difference between the steady-state capital stock and the debt-tolerance parameter $\bar{a}$. The conditions in (14) permit, nevertheless, $\tilde{n}$ and $\tilde{a}$ to be negative.[13] What is the effect of an increase in the status parameter β on the steady-state values of $\tilde{c}$, $\tilde{n}$, and $(\tilde{a} - \bar{a})$? Using the solutions (13a)-(13c), it is straightforward to show that higher values of β—corresponding to a higher degree of status-

[13] Obviously, if the signs of both conditions in (14) are reversed, then $\tilde{c}$ and $(\tilde{a} - \bar{a})$ are still positive. In this case, however, the solution to the homogenous system $(\dot{c}, \dot{n})$ described below in equation (20) does not possess saddlepoint dynamics.

consciousness—lead to greater values of $\tilde{c}$, $\tilde{n}$, and $(\tilde{a} - \bar{a})$.[14] We can also employ the following numerical parameterization of the economy, assuming Cobb-Douglas technology, that satisfies the restrictions stated in (14):[15]

$$\rho = 0.04, \quad r^* = 0.03, \quad \eta = 0.8, \quad \bar{a} = -0.5\tilde{y}, \quad y = k^{0.3}, \quad \epsilon = 1. \tag{15a}$$

Since the steady–state marginal product of capital equals the world interest rate according to equation (10c), the steady-state values of the domestic capital stock and output correspond to:

$$\tilde{k} = 26.827, \qquad \tilde{y} = 2.6827. \tag{15b}$$

Using (15a)-(15b) and the solutions for $\tilde{c}$, $\tilde{n}$, and $(\tilde{a} - \bar{a})$ given in (13a)-(13c), we can calculate the effects of greater values of β on the steady-state of the consumption-side of the economy. These results are stated in Table 1, where we permit the values of the status-parameter β to range from 0.04 to 0.24.[16] Observe that if β takes the relatively "low" value of 0.04, the economy is a net debtor in steady state equilibrium, $(\tilde{n} = -17.3587)$. In this case agents' preference for relative wealth is insufficiently strong to induce agents to become net creditors in long-run equilibrium.[17] Nevertheless, in accordance with the restrictions in (14), net assets in steady-state equilibrium still exceed their lower bound of $\bar{a}$, $[(a - \bar{a}) = 10.8097]$. As β assumes progressively higher values, ($\beta = 0.12$, 0.16, 0.20, 0.24), we see from Table 1 that the small open economy becomes a greater net creditor in long-run equilibrium. Since a greater stock of international assets also implies more net interest income, higher values of β permit agents to enjoy increasing levels of steady-state

[14]We can also demonstrate that an improvement in total factor productivity ϵ increases $\tilde{c}$, $\tilde{n}$, and $(\tilde{a} - \bar{a})$, while a rise in the world interest rate r^* has ambiguous effects on the levels of these steady state variables.

[15]We have attempted to choose parameter values—consistent with our structural model—in line with those used by authors such as Lucas (1988), Jones (1995), Ortigueira and Santos (1997), and Ogaki and Reinhart (1998). Below, we specify two values for the effective intertemporal elasticity of substitution: $\sigma^e = 0.4$ and $\sigma^e = 2.5$. The former value is closer to empirical estimates of the intertemporal elasticity of substitution.

[16]The values chosen for η and β will satisfy the conditions on instantaneous preferences in equation (3), given the values of θ that we will assume below. For the values of ρ, r^*, and η stated in (15a), the implied upper-bound on β, consistent with the first condition of (14) is 0.2667. All numerical simulations in this paper were done using *Mathematica* 4.1.

[17]For the parameterization given in (15a)-(15b), we can show that the critical value of β for which $\tilde{n} = 0$ is 0.084.

consumption $\tilde{c}$, as is evident from the first column of Table 1 in which $\tilde{c}$ increases from 2.1619 to 18.3765 as β rises from 0.04 to 0.24.

The next step is to linearize the differential equations for $(\dot{c},\, \dot{q},\, \dot{k},\, \dot{n})$ about the steady-state equilibrium described above. This procedure yields the following relationships

$$\dot{c} = \frac{\sigma^e\,(\beta/\eta)\,\tilde{c}}{\tilde{n}+\tilde{k}-\bar{a}}(c-\tilde{c}) - \frac{\sigma^e\,(\beta/\eta)\,\tilde{c}^2}{(\tilde{n}+\tilde{k}-\bar{a})^2}(k-\tilde{k}) - \frac{\sigma^e\,(\beta/\eta)\,\tilde{c}^2}{(\tilde{n}+\tilde{k}-\bar{a})^2}(n-\tilde{n})$$

$$= \sigma^e\,(\rho - r^*)\,(c-\tilde{c}) - \sigma^e\,(\rho - r^*)^2\,(\beta/\eta)^{-1}\left[(k-\tilde{k}) + (n-\tilde{n})\right], \tag{16a}$$

$$\dot{q} = \left[r^* + \frac{(\beta/\eta)\,\tilde{c}}{\tilde{n}+\tilde{k}-\bar{a}}\right](q-1) - \epsilon f''(\tilde{k})(k-\tilde{k})$$

$$= \rho\,(q-1) - \epsilon f''(\tilde{k})(k-\tilde{k}), \tag{16b}$$

$$\dot{k} = h^{-1}\tilde{k}(q-1), \tag{16c}$$

$$\dot{n} = -(c-\tilde{c}) - h^{-1}\tilde{k}(q-1) + r^*(k-\tilde{k}) + r^*(n-\tilde{n}). \tag{16d}$$

where we have used the fact from (13a)-(13c) that $\tilde{c}/(\tilde{a}-\bar{a}) = (\beta/\eta)^{-1}\,(\rho - r^*)$ to derive the second equalities of (16a)-(16b). Observe that the differential equations (16b)-(16c) constitute an two-equation dynamic system in $(\dot{q}, \dot{k})$ that can be solved independently of (16a)-(16d), the differential equations for consumption and the current account balance.[18]

3.2. Production-Side Dynamics

Re-expressing (16b)-(16c) as a matrix system, we state the production-side dynamics:

$$\begin{pmatrix} \dot{q} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} \rho & -\epsilon f''(\tilde{k}) \\ h^{-1}\tilde{k} & 0 \end{pmatrix} \begin{pmatrix} q-1 \\ k-\tilde{k} \end{pmatrix}. \tag{17}$$

[18]This will not be the true if the employment decision were endogenous. In that case the evolution of capital stock and its shadow value depends on the Euler equation and the current account balance.

The dynamic and stability properties of (17) can be determined by examining its characteristic equation, which is equal to

$$0 = -(\rho - \mu)\,\mu + h^{-1}\tilde{k}\epsilon f''(\tilde{k}) = \mu^2 - \mathrm{tr}\,(\mathbf{J})\,\mu + \det(\mathbf{J})\,, \tag{17}$$

where the trace and determinant of the Jacobean matrix $\mathbf{J}$ in (17) are given by:

$$\mathrm{tr}\,(\mathbf{J}) = \mu_1 + \mu_2 = \rho > 0, \quad \det(\mathbf{J}) = \mu_1\mu_2 = h^{-1}\tilde{k}\epsilon f''(\tilde{k}) < 0.$$

Since $\det(\mathbf{J}) < 0$, the steady state equilibrium $(\tilde{k}, 1)$ describes a saddlepoint, with the stable and unstable eigenvalues having the following properties: $\mu_1 < 0$, $\mu_2 > 0$, $|\mu_1| < \mu_2$. Employing standard techniques, we can calculate the following stable solution for the domestic capital stock and its shadow value

$$q - 1 = \mu_1 h\tilde{k}^{-1}(k - \tilde{k}) = \frac{\epsilon f''(\tilde{k})}{\rho - \mu_1}(k - \tilde{k}), \tag{18a}$$

$$k = \tilde{k} - (\tilde{k} - k_0)e^{\mu_1 t}, \tag{18b}$$

such that the domestic capital stock adjusts from a given initial value, k_0.[19] The dynamics of this system is illustrated in Figure 1, which depicts the $\dot{q} = 0$ locus, the $\dot{k} = 0$ locus (equal to $\tilde{q} = 1$), the stable saddlepath XX, and the steady-state equilibrium $(\tilde{k}, 1)$ corresponding to point D. Because the slope of the $\dot{q} = 0$ locus is given by $(dq/dk)\,|_{\dot{q}=0} = \epsilon f''(\tilde{k})/\rho < 0$, it is clear that:

$$\left.\frac{dq}{dk}\right|_{XX} = \frac{\epsilon f''(\tilde{k})}{\rho - \mu_1} > \left.\frac{dq}{dk}\right|_{\dot{q}=0} = \frac{\epsilon f''(\tilde{k})}{\rho}.$$

In other words, the $\dot{q} = 0$ locus is "steeper" that the saddlepath XX.[20] Since the stable saddlepath XX is negatively sloped, the capital stock and its shadow value move in

[19]See Turnovsky (2000) for a recent exposition of these methods and Blanchard and Fischer (1989), chapter 2, for a description of the Tobin's q framework. Note that we are implicitly imposing the transversality condition for physical capital in order to eliminate the part of the solution that contains the positive eigenvalue μ_2. We follow the same procedure below in solving the $(\dot{c}, \dot{n})$ system.

[20]The slope of $\dot{q} = 0$ is unambiguously negative, since it is evaluated in the steady-state equilibrium in which $\tilde{q} = 1$ and $\left[(\beta/\eta)\,\tilde{c}/(\tilde{n} + \tilde{k} - \bar{a})\right] = \rho - r^*$.

opposite directions, i.e., $sgn\ (\dot{q}) = sgn\ (\dot{k})$. In Figure 1 we have also indicated at point B the initial position of the production-side of the economy as well as the arrows determining the direction of movement in the phase plane. Since $q\,(t) > 1$ between points B and D, physical investment is positive, $\dot{k}\,(t) > 0$, during the entire transition to long-run equilibrium. Solving the characteristic equation (17) for stable eigenvalue μ_1, we obtain:

$$\mu_1 = \frac{1}{2}\left\{\mathrm{tr}\,(\mathbf{J}) - \sqrt{[\mathrm{tr}\,(\mathbf{J})]^2 + 4\,|\det\,(\mathbf{J})|}\right\} = \frac{1}{2}\left\{\rho - \sqrt{\rho^2 - 4h^{-1}\tilde{k}\epsilon f''(\tilde{k})}\right\}. \tag{19a}$$

This expression reveals that speed of stable adjustment of the production-side of the economy—equal to $|\mu_1|$—is a function of the rate of time preference ρ and of the curvature properties of the installation cost function $\Psi\,(i,k)$ and the production function $f\,(k)$, in addition to total factor productivity ϵ.[21] It is, furthermore, independent of the parameters of the instantaneous utility function described above in equation (3). By direct calculation, we can show the following relationships between $|\mu_1|$ and ρ, ϵ, $f''(\tilde{k})$, and h obtain:

$$\frac{\partial\,|\mu_1|}{\partial\rho} \gtrless 0, \quad \frac{\partial\,|\mu_1|}{\partial\epsilon} > 0, \quad \frac{\partial\,|\mu_1|}{\partial[f''(\tilde{k})]} < 0, \quad \frac{\partial\,|\mu_1|}{\partial h} < 0. \tag{19b}$$

The partial derivatives in (19b) imply: i) an ambiguous relationship between the rate of time preference ρ and $|\mu_1|$, ii) a positive relationship between total factor productivity ϵ and $|\mu_1|$, iii) a negative relationship between the curvature of the production function $f''(\tilde{k})$ and $|\mu_1|$, and iv) a negative relationship between the installation cost function parameter h and $|\mu_1|$. In Table 2a we illustrate—using our numerical parameterization in (15a)-(15b)—the negative relationship between h and $|\mu_1|$ for values of h ranging from 1 to 20. Clearly, higher values of the adjustment cost parameter h lower the speed of adjustment $|\mu_1|$. We will use these results subsequently in analyzing the dynamics of the current account balance.

[21] In particular, note that the term $h^{-1}\tilde{k}$ corresponds to $\left[\partial^2\Psi(i,k)/\partial i^2\right]^{-1}$.

3.3. Consumption-Side Dynamics

Our next step is to use the production-side solutions (18a)-(18b) to determine the paths of consumption and the current account. Substituting (18a)-(18b) into the linearized expressions for $\dot{c}$ and $\dot{n}$ in (15a)-(15d), we obtain the following nonhomogeneous differential equation system in $(\dot{c}, \dot{n})$

$$\begin{pmatrix} \dot{c} \\ \dot{n} \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} \\ -1 & r^* \end{pmatrix} \begin{pmatrix} c - \tilde{c} \\ n - \tilde{n} \end{pmatrix} + \begin{pmatrix} -d_{12} \\ -(r^* - \mu_1) \end{pmatrix} \left(\tilde{k} - k_0\right) e^{\mu_1 t}, \tag{20}$$

where:

$$d_{11} = \sigma^e (\rho - r^*) > 0, \quad d_{12} = -\sigma^e (\rho - r^*)^2 (\beta/\eta)^{-1} < 0.$$

The complete solution of (20) is the sum of the general solution of the homogenous system and a particular solution of the nonhomogeneous part of (20). Employing the same methods used to solve (16), it is easy to show that the stable solution to the homogenous part of (20) can be expressed as

$$c = \tilde{c} + (r^* - \omega_1) A_1 e^{\omega_1 t}, \tag{21a}$$

$$n = \tilde{n} + A_1 e^{\omega_1 t}, \tag{21b}$$

where $\omega_1 < 0$ is the stable eigenvalue of the Jacobian $\mathbf{G}$ of homogenous part of (20) and where A_1 is an arbitrary constant to be determined. From the Jacobian matrix $\mathbf{G}$, it is straightforward to show that the characteristic equation of $\mathbf{G}$ is equal to

$$0 = (d_{11} - \omega)(r^* - \omega) + d_{12} = \omega^2 - \mathrm{tr}(\mathbf{G})\, \omega + \det(\mathbf{G}), \tag{22}$$

where the trace and determinant of the Jacobian of (20) correspond to:

$$\mathrm{tr}(\mathbf{G}) = \omega_1 + \omega_2 = d_{11} + r^* = \sigma^e (\rho - r^*) + r^* > 0,$$

$$\det(\mathbf{G}) = \omega_1 \omega_2 = d_{11} r^* + d_{12} = -\sigma^e (\rho - r^*) (\beta/\eta)^{-1} [\rho - [1 + (\beta/\eta)] r^*] < 0.$$

The expressions for tr($\mathbf{G}$) and det($\mathbf{G}$) imply that the eigenvalues of $\mathbf{G}$, given our assumptions, obey the following relationships: $\omega_1 < 0$, $\omega_2 > 0$, $|\omega_1| < \omega_2$. Note that the condition $\rho - r^* > 0$ is sufficient for $\mathrm{tr}(\mathbf{G}) > 0$. In contrast, $\det(\mathbf{G}) < 0$ is ensured only if both $\rho - r^* > 0$ and the first restriction in (14) hold. If $\det(\mathbf{G}) < 0$, the homogenous part of the solution of (20) displays saddlepoint dynamics. Using the characteristic equation (22), we can write the expression for the stable eigenvalue ω_1 as:

$$\omega_1 = \frac{1}{2}\left\{\mathrm{tr}\left(\mathbf{G}\right) - \sqrt{\left[\mathrm{tr}\left(\mathbf{G}\right)\right]^2 + 4\left|\det\left(\mathbf{G}\right)\right|}\right\}$$

$$= \frac{1}{2}\left\{\sigma^e\left(\rho - r^*\right) + r^* - \sqrt{\left[\sigma^e\left(\rho - r^*\right) + r^*\right]^2 + 4\sigma^e\left(\rho - r^*\right)\left(\beta/\eta\right)^{-1}\left[\rho - \left[1 + \left(\beta/\eta\right)\right]r^*\right]}\right\} \tag{23}$$

Clearly, the speed of stable adjustment $|\omega_1|$ of the homogenous system depends on the value of the world interest rate r^* and preference parameters such as the rate of time discount ρ, the effective intertemporal elasticity of substitution $\sigma^e = [1 - \eta(1-\theta)]^{-1}$, and the term β that measures the importance of relative wealth for status-conscious consumers. Observe, in addition, that $|\omega_1|$ depends neither on the characteristics of the production function nor on the installation cost function. Employing the above numerical parameterization for specified values of σ^e, we can consider the effect of higher values of β on the stable speed of adjustment.[22] We perform the calculations for two values of the effective intertemporal elasticity of substitution, $\sigma^e = 0.4$ and $\sigma^e = 2.5$, and, as before, allow the status-parameter β to range from 0.04 to 0.24.[23] The numerical expressions for $|\omega_1|$ are reported in Table 2b and reveal that *higher* values of β lead to *reductions* in $|\omega_1|$. In other words, the speed of adjustment of the consumption-side economy "slows down" the more important are status considerations, a result that is robust, according to Table 2b, regardless of whether σ^e is less than ($\sigma^e = 0.4$) or greater than ($\sigma^e = 2.5$) unity.[24] Indeed, the

[22]Concerning the other parameters, it is straightforward to show analytically that the relationships between $|\omega_1|$ and ρ, r^*, and σ^e, are, in general, ambiguous.

[23]Given the expression for σ^e in (9a) the corresponding values of θ for $\eta = 0.8$ are, respectively, $\theta = 2.875$ if $\sigma^e = 0.4$ and $\theta = 0.25$ if $\sigma^e = 2.5$. Clearly, our chosen values of θ, η, and β satisfy the parameter restrictions on the instantaneous utility function (3).

[24]In our parameterization, $|\omega_1|$ is higher for larger values of σ^e, given β.

speed of adjustment declines by a factor of 35.69 as β rises from 0.04 to 0.24 if $\sigma^e = 0.4$, while it falls by a factor of 28.22 as β rises from 0.04 to 0.24 if $\sigma^e = 2.5$.

Continuing, we specify that a particular solution of (20) corresponds to the following equations

$$c = \tilde{c} + E_1 e^{\mu_1 t}, \tag{24a}$$

$$n = \tilde{n} + F_1 e^{\mu_1 t}, \tag{24b}$$

where E_1 and F_1 are given constants to be determined and μ_1 is the stable eigenvalue of the production-side of the economy. Substitution of the particular solutions (24a)-(24b) into the nonhomogeneous dynamic system (20), yields $E_1 = 0$ and $F_1 = (\tilde{k} - k_0)$, which implies that the particular solutions (24a)-(24b) for consumption and net financial assets become:

$$c = \tilde{c}, \quad n = \tilde{n} + (\tilde{k} - k_0) e^{\mu_1 t}. \tag{25a-25b}$$

Combining the solutions (21a)-(21b) of the homogenous system with the particular solutions (25a)-(25b), we obtain the following general solution of the nonhomogeneous system:

$$c = \tilde{c} + (r - \omega_1) A_1 e^{\omega_1 t} \tag{26a}$$

$$n = \tilde{n} + A_1 e^{\omega_1 t} + (\tilde{k} - k_0) e^{\mu_1 t}. \tag{26b}$$

The arbitrary constant A_1 is then found using the assumption that the stock of net financial assets adjusts from an initial value, i.e., $n(0) = n_0$. This implies $A_1 = -\left[(\tilde{n} - n_0) + (\tilde{k} - k_0)\right] = -(\tilde{a} - a_0)$. Consequently, the complete solutions for consumption and net financial assets equal:

$$c = \tilde{c} - (r^* - \omega_1)\left[(\tilde{n} - n_0) + (\tilde{k} - k_0)\right] e^{\omega_1 t}, \tag{27a}$$

$$n = \tilde{n} - \left[(\tilde{n} - n_0) + (\tilde{k} - k_0)\right] e^{\omega_1 t} + (\tilde{k} - k_0)e^{\mu_1 t}. \tag{27b}$$

3.4. Current Account Dynamics

To review our results thus far, we have showed that the small open economy with status-preferences for relative wealth and installation costs of accumulating physical capital possesses two distinct speeds of adjustment, $|\mu_1|$ and $|\omega_1|$. The dynamics of the capital stock and its shadow value depend solely on $|\mu_1|$, while that of consumption depends solely on $|\omega_1|$. Inspection of the solution for net financial assets in (27b) shows, however, that it depends on both $|\mu_1|$ and $|\omega_1|$. Taking the time derivative of equation (27b)—which converts it into an expression for the current account balance—we obtain:

$$\dot{n}\,(t) = -\omega_1 \left[(\tilde{n} - n_0) + (\tilde{k} - k_0)\right] e^{\omega_1 t} + \mu_1(\tilde{k} - k_0)e^{\mu_1 t}. \tag{28}$$

Assuming that net financial assets and physical capital at $t = 0$ are below their steady-state values, i.e., $(\tilde{n} - n_0) > 0$, $(\tilde{k} - k_0) > 0$, the expression for $\dot{n}(t)$ in (28) reveals that the two speeds of adjustment have opposite effects on $\dot{n}(t)$.[25] That is, the first term on the right-hand-side of (28) dependent on $|\omega_1|$ is associated with an *improvement* in the current account $[\dot{n}\,(t) > 0]$, while the second term on the right-hand-side of (28) dependent on $|\mu_1|$ is associated with a *deterioration* in the current account $[\dot{n}\,(t) < 0]$. Clearly, then, the dynamics of the current account balance depend on the relative sizes of $|\mu_1|$ and $|\omega_1|$ and, consequently, on the relative magnitudes of the parameters that influence consumption and production-side dynamics.

To investigate in more detail the conditions under the influence of either $|\omega_1|$ or $|\mu_1|$ dominates, we must determine whether there exists a time $t^* > 0$ such that the stock of international assets reaches a stationary value during its transitional adjustment, i.e.,

[25] In terms of a comparative statics experiment, this would correspond to the case in which a permanent increase in ϵ (a positive productivity shock) results—starting from (n_0, k_0)—in an increase $\tilde{n}$ and $\tilde{k}$. Of course, the initial value of consumption, $c\,(0)$, will be chosen consistent with the economy's transversality conditions.

$\dot{n}(t^*) = 0$. Solving (28) for t^*, we obtain

$$t^* = -\frac{\ln\left[\frac{-\omega_1(\tilde{n}-n_0)}{(\omega_1-\mu_1)(\tilde{k}-k_0)}\right]}{(\omega_1-\mu_1)} \tag{29a}$$

where the necessary conditions for $t^* > 0$ correspond to:

$$t^* > 0 \quad \Longleftrightarrow \quad 0 < \left[\frac{-\omega_1(\tilde{n}-n_0)}{(\omega_1-\mu_1)(\tilde{k}-k_0)}\right] < 1, \quad (\omega_1-\mu_1) > 0. \tag{29b}$$

If the conditions in (29b) hold, the current account will initially deteriorate $[\dot{n}(t) < 0, 0 \leq t < t^*]$ before reaching a stationary value at time t^*. Subsequently, the current account improves $[\dot{n}(t) > 0, t > t^*]$ as the stock of international assets rises to its higher steady state level. The latter condition in (29b) is, of course, equivalent to $|\mu_1| > |\omega_1|$, i.e., that the speed of adjustment of the production-side of the economy exceeds the speed of adjustment of the consumption-side of the economy. Observe, in addition, that the first condition in (29b) for the existence of $\dot{n}(t^*) = 0$ depends not only on the relative sizes of $|\mu_1|$ and $|\omega_1|$, but also on the ratio of the changes in the stocks of net assets and physical capital, $[(\tilde{n}-n_0)/(\tilde{k}-k_0)]$. If, instead, the conditions in (29b) are violated, then the path of $n(t)$ will not reach a stationary value during the transition to steady-state equilibrium. In this case, the current account improves $[\dot{n}(t) > 0, t \geq 0]$ during the entire phase of adjustment starting from $n(0) = n_0$.

In light of our previous discussion, the question naturally arises how agents' status-consciousness influences the conditions in (29b) and, consequently, the transitional behavior of the current account. On the one hand, an increase in β lowers $|\omega_1|$, which puts downward pressure on the current account balance. On the the other hand, higher values of β also lead to greater long-run stocks of net international financial assets $\tilde{n}$, which, of course, requires the accumulation of current account surpluses. In general, then, a greater degree of status-consciousness has an ambiguous effect on the dynamics of the current account. We can, however, analyze the cases in which (29b) holds for our numerical parameterization of the economy. Specifically, we can use our results from Tables 1, 2a, and

2b to calculate the term

$$\frac{-\omega_1(\tilde{n}-n_0)}{(\omega_1-\mu_1)(\tilde{k}-k_0)}$$

which we denote by Δ. Our calculations for Δ are given in Tables 3a and 3b. In Table 3a we calculate Δ for the case $\sigma^e = 0.4$, while in Table 3b we set $\sigma^e = 2.5$. In both tables, we permit the status parameter β to range from 0.12 to 0.24 and the adjustment cost parameter h to assume values from 1 to 20.[26] Note that we exclude $\beta = 0.04$ and $\beta = 0.08$ in Tables 3a and 3b, since we only wish to consider cases in which $\tilde{n} > 0$ and $|\mu_1| > |\omega_1|$. Inspection of Table 3a reveals that $0 < \Delta < 1$ in all cases and, consequently, that the necessary conditions in (29b) for $t^* > 0$ hold. This implies that $n(t)$ attains a stationary value during its adjustment phase. In contrast, these conditions need not obtain if σ^e equals the higher value of 2.5. If the adjustment cost parameter takes relatively low values, $h = (1, 5)$, then $0 < \Delta < 1$ and the current account will reach a stationary value at time t^*. On the other hand, if h assumes relatively high values, $h = (15, 20)$, then $\Delta > 1$, which implies a positive current account balance $[\dot{n}(t) > 0]$ during the entire transitional phase. In the intermediate case in which $h = 10$, we calculate $0 < \Delta < 1$ for $\beta = (0.12,$ $0.16, 0.20)$, but $\Delta > 1$ if $\beta = 0.24$. In the latter case, the relatively high value of the status parameter β implies a relatively large change in the stock of long-run international assets $\tilde{n}$ (see Table 1), which requires that the economy run current account surpluses for $t \geq 0$. This is true despite the fact that the consumption-side speed of adjustment $|\omega_1|$ (see Table 2b) is relatively low in this case.

We can also illustrate our results by means of phase diagrams.[27] Figure 2a illustrates in the case in which the conditions in (29b) hold, while Figure 2b depicts the case in which they are violated.[28] As illustrated in Figure 2a, the current account at $t = 0$ deteriorates, $[\dot{n}(0) < 0]$. This decline in the current account balance reflects the relatively rapid adjustment of the physical capital stock compared to that of the stock of international assets. We can identify two reasons for this response. As indicated, one is the fact that the speed of adjustment of the production-side of the economy is greater that of the consumption-side,

[26]To calculate Δ in Tables 3a, 3b we set, without loss of generality, $(\tilde{k}-k_0) = 26.827$. Similarly, $(\tilde{n}-n_0)$ equals the values of $\tilde{n}$ calculated for $\beta = (0.12, 0.16, 0.20, 0.24)$ in Table 1.

[27]In Figures 2 we assume $(\tilde{n}-n_0) > 0$.

[28]If only the second condition in (29b) holds, while the first condition is violated, then the current account balance is positive $(\dot{n}(t) > 0)$ during the entire adjustment phase.

$|\mu_1| > |\omega_1|$. Intuitively, this implies a relatively large initial "build-up" of adjustment costs of investment, which places downward pressure on the current account balance. The other condition for this result requires the ratio $[(\tilde{k}-k_0)/(\tilde{n}-n_0)]$ to exceed a critical level, determined by rewriting the first condition in (29b).[29] After $t=0$, the economy proceeds along the locus LMN in its transition to the steady-state equilibrium at point N. Observe that consumption continuously rises along the locus LMN. This reflects the on-going increase in output due to the rising stock of capital. At point M, determined by solution for $\dot{n}(t^*)=0$, where t^* is given by (29a), the stationary, and minimum, value of $n(t^*)$ is reached. Thereafter, for $t>t^*$, both the stock of net financial assets and consumption increase toward $(\tilde{c},\tilde{n})$.[30] This reflects the relative reduction in the rate of physical capital accumulation compared to that of net financial assets as $t\to\infty$. Observe in Figure 2a that the slope of the LMN locus approaches $(r^*-\omega_1)>0$ as the path of (c,n) approaches point N. Using the solutions in (27a, b), this is derived by calculating the following ratio:

$$\frac{(c-\tilde{c})}{(n-\tilde{n})} = \frac{-(r^*-\omega_1)\left[(\tilde{n}-n_0)+(\tilde{k}-k_0)\right]}{-\left[(\tilde{n}-n_0)+(\tilde{k}-k_0)\right]+(\tilde{k}-k_0)e^{(\mu_1-\omega_1)t}}. \tag{30}$$

Clearly, since $|\mu_1|>|\omega_1|$ in this case, we obtain:

$$\text{slope } \frac{(c-\tilde{c})}{(n-\tilde{n})} \to (r^*-\omega_1), \text{ as } t\to\infty.$$

[29] ¿From (29b), this condition is satisfied if:

$$\frac{(\tilde{k}-k_0)}{(\tilde{n}-n_0)} > \frac{-\omega_1}{(\omega_1-\mu_1)}, \text{ where } (\omega_1-\mu_1)>0.$$

As discussed above, higher values of the status parameter β cause both $(\tilde{k}-k_0)/(\tilde{n}-n_0)$ and $-\omega_1/(\omega_1-\mu_1)$ to decline.

[30] In deriving Figures 2a and 2b, we used the fact that slope of the (n,c) locus is equal to

$$\frac{dc}{dn} = \frac{dc/dt}{dn/dt} = \frac{\dot{c}(t)}{\dot{n}(t)}$$

while the curvature of the locus is given by:

$$\frac{d^2c}{dn^2} = \frac{\frac{d^2c}{dt^2}\frac{dt}{dn}\frac{dn}{dt} - \frac{dc}{dt}\frac{d^2n}{dt^2}\frac{dt}{dn}}{(dn/dt)^2} = \frac{\ddot{c}(t)\dot{n}(t)-\dot{c}(t)\ddot{n}(t)}{\ddot{n}(t)}.$$

We turn next to Figure 2b, which illustrates the case in which one or both of the conditions in (29b) are violated. The adjustment of net financial assets and consumption in Figure 2b is depicted by the locus PQ. Along PQ, the current account balance is always positive and agents enjoy increasing levels of consumption, i.e., $\dot{n}(t) > 0$, $\dot{c}(t) > 0$. While installation costs of investment are positive in this case, its effect on the domestic demand is insufficient to cause the current account balance to initially deteriorate during the transition to the new steady state at point Q. We graph in Figure 2b the case in which the slope of the PQ locus approaches zero as (c, n) approach their long-run values at point Q. Recalling equation (30), this holds if $|\omega_1| > |\mu_1|$, i.e.:[31]

$$\text{slope } \frac{(c - \tilde{c})}{(n - \tilde{n})} \rightarrow 0, \text{ as } t \rightarrow \infty.$$

Before we leave this section, it is important to discuss a key difference between Figure 1, the phase diagram for the capital stock and its shadow value, and Figures 2a and 2b that describe the dynamics of consumption and the current account balance. In particular, note in Figures 2 that we have illustrated neither the $\dot{n} = 0$ and $\dot{c} = 0$ loci nor the arrows of motion of the (n, c) phase plane. Recalling the expressions for $\dot{n}(t)$ and $\dot{c}(t)$ in equations (7e) and (8), observe that they depend on the value of the capital stock k at time t. Since, however, the capital stock is a dynamic variable that evolves continuously, the positions of the $\dot{n} = 0$ and $\dot{c} = 0$ loci—as well as the directional arrows in the phase plane—are also continually shifting through time. Consequently, they cannot be drawn in Figures 2a and 2b.

4. Conclusions

As is well-known, the basic small open economy Ramsey model has two problematic characteristics: i) the lack of an interior equilibrium if the rate of time preference differs from the world interest rate and ii) no investment dynamics if the marginal physical product of capital is always equal to the exogenous and time invariant world interest rate. Incorporating preferences over relative wealth can overcome the first problem. Nevertheless, the

[31] If the second condition of (29b) is satisfied, i.e., $|\mu_1| > |\omega_1|$, while the first condition is violated, then then slope of locus PQ approaches $(r^* - \omega_1)$ as $t \rightarrow \infty$.

second issue cannot be resolved with relative wealth preferences alone. The purpose of this paper was to investigate the intertemporal equilibrium of a small open economy that combines both relative wealth preferences and capital stock dynamics.

Among our results, we found that in our framework the dynamics of small open economy depends on two distinct speeds of adjustment: one arising from the status-consciousness and the other deriving from the process of accumulating physical assets. We showed, nevertheless, that the dynamics of the current account balance depends on both speeds of adjustment. In fact, the two speeds of adjustment tend to have opposing short-run effects on the current account. A larger production-side speed of adjustment leads initially to current account deficits, while a larger consumption-side speed of adjustment results in current account surpluses in the short-run as well as in the long-run. Consequently, the current account balance can exhibit non-monotonic paths. For instance, the current account balance can initially deteriorate before improving in the transition to the steady-state equilibrium.

In analyzing in greater detail the dynamics of international assets, we derived the conditions that determine whether it reaches a stationary value *prior* to steady-state equilibrium. These conditions depend on the speeds of adjustment as well as on the long-run dynamics of international assets and domestic physical capital. We found that because the degree of status-consciousness both *lowers* the speed of adjustment of the consumption-side of the economy and *raises* the stock of international assets, its relationship to the evolution of the current account is complex. In this context, we conducted a numerical analysis to examine how the degree of status-consciousness affects the dynamics of the current account under different parameter values.

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Table 1

The Effect of β on the Steady-State Equilibrium

	$\tilde{c}$	$\tilde{n}$	$(\tilde{a} - \bar{a})$
$\beta = 0.04$	2.1619	-17.3587	10.8097
$\beta = 0.08$	2.6252	-1.9163	26.2521
$\beta = 0.12$	3.3412	21.9493	50.1177
$\beta = 0.16$	4.5941	63.714	91.8824
$\beta = 0.20$	7.3506	155.596	183.7648
$\beta = 0.24$	18.3765	523.126	551.2944

Table 2a

Solutions for $|\mu_1|$

$h = 1$	$\lvert\mu_1\rvert = 0.126$
$= 5$	$= 0.04771$
$= 10$	$= 0.02993$
$= 15$	$= 0.02237$
$= 20$	$= 0.01803$

Table 2b

Solutions for $|\omega_1|$

	$\sigma^e = 0.4$	$\sigma^e = 2.5$
$\beta = 0.04$	$\lvert\omega_1\rvert = 0.01431$	$\lvert\omega_1\rvert = 0.04325$
$= 0.08$	$= 0.00685$	$= 0.02256$
$= 0.12$	$= 0.00387$	$= 0.0134$
$= 0.16$	$= 0.00221$	$= 0.00794$
$= 0.20$	$= 0.00114$	$= 0.00422$
$= 0.24$	$= 00039$	$= 0.00148$

Table 3a

Value of Δ for $\sigma^e = 0.4$

	$h = 1$	$h = 5$	$h = 10$	$h = 15$	$h = 20$
$\beta = 0.12$	$\Delta = 0.02593$	$\Delta = 0.07225$	$\Delta = 0.01215$	$\Delta = 0.17115$	$\Delta = 0.22361$
$= 0.16$	$= 0.0424$	$= 0.11536$	$= 0.18935$	$= 0.26035$	$= 0.33178$
$= 0.20$	$= 0.05296$	$= 0.14199$	$= 0.22966$	$= 0.31145$	$= 0.39147$
$= 0.24$	$= 0.06054$	$= 0.25745$	$= 0.25745$	$= 0.346$	$= 0.43112$

Table 3b

Value of Δ for $\sigma^e = 2.5$

	$h = 1$	$h = 5$	$h = 10$	$h = 15$	$h = 20$
$\beta = 0.12$	$\Delta = 0.09737$	$\Delta = 0.31955$	$\Delta = 0.66325$	$\Delta = 1.22225$	$\Delta = 2.36795$
$= 0.16$	$= 0.15973$	$= 0.47416$	$= 0.85755$	$= 1.30682$	$= 1.86893$
$= 0.20$	$= 0.20095$	$= 0.56254$	$= 0.95126$	$= 1.34705$	$= 1.76977$
$= 0.24$	$= 0.23177$	$= 0.62427$	$= 1.01441$	$= 1.38152$	$= 1.7438$

Figure 1

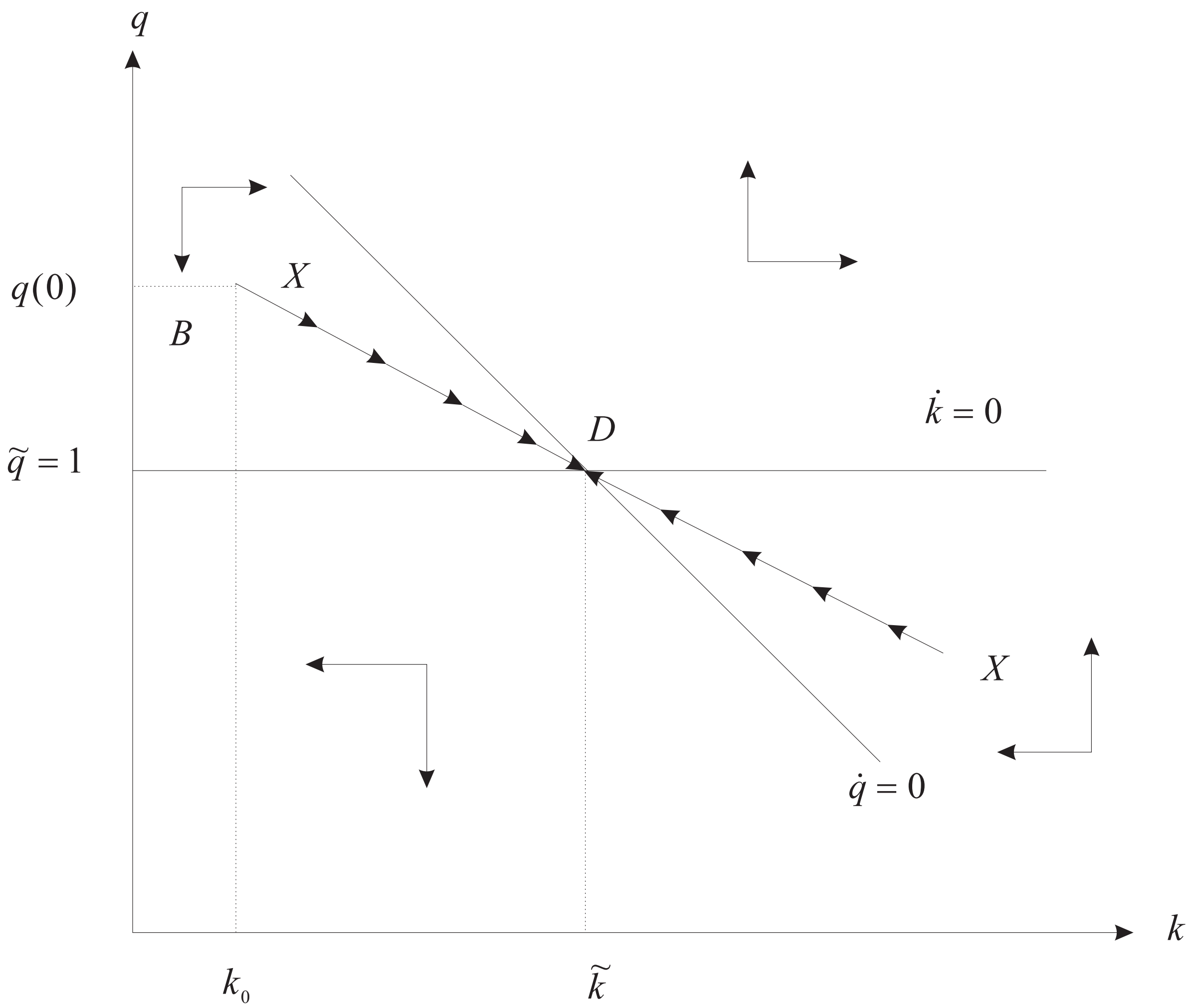


Figure 2a

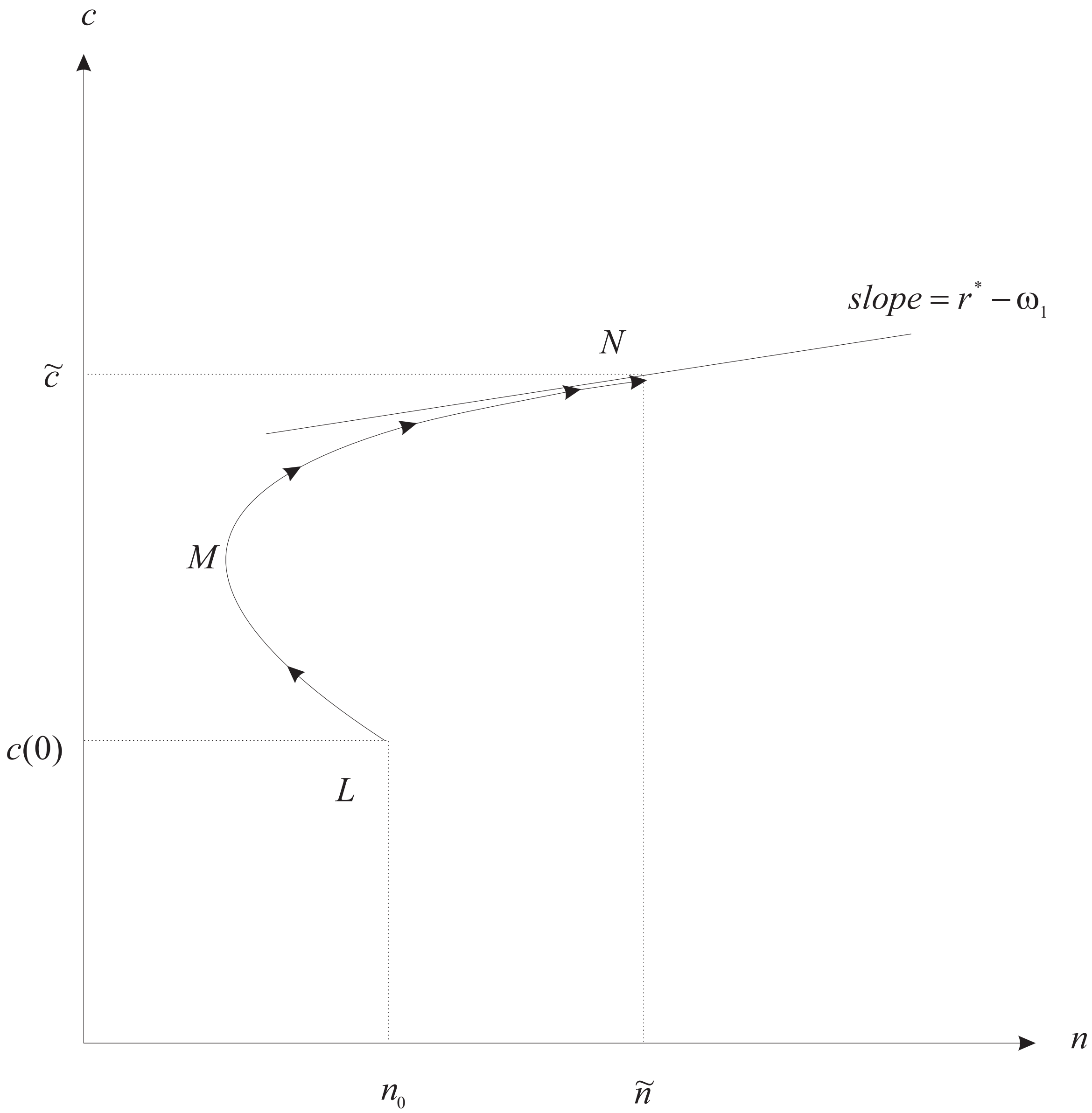
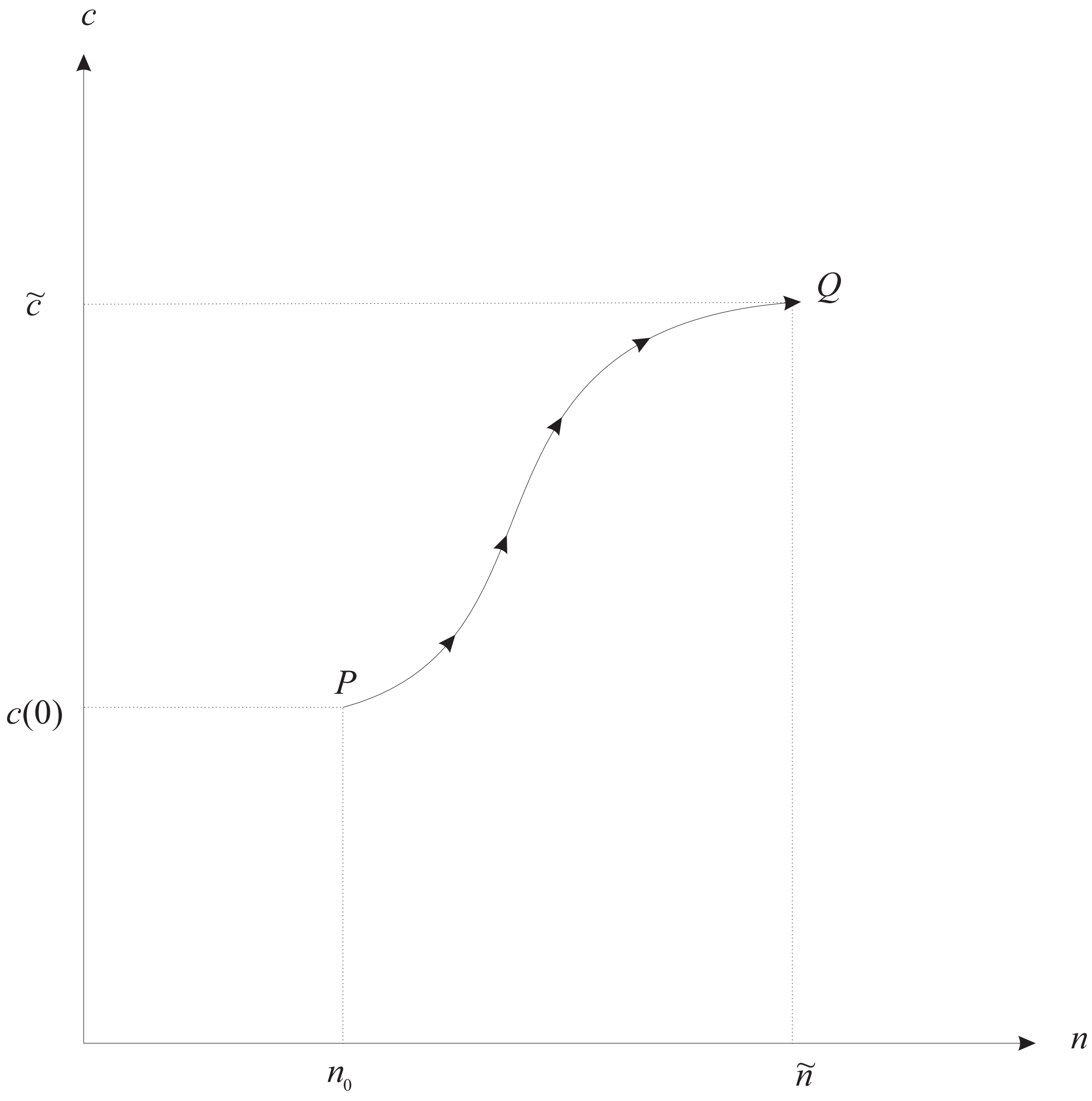


Figure 2b



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Title: Investment and Current Account Dynamics in an Open Economy Status Seeking Framework

Reihe Ökonomie / Economics Series 110

ISSN: 1605-7996

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ISSN: 1605-7996